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Black Hole Attractors in Extended Supergravity

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Abstract

We review some aspects of the attractor mechanism for extremal black holes of (not necessarily supersymmetric) theories coupling Einstein gravity to scalars and Maxwell vector fields. Thence, we consider $\mathcal{N} = 2$ and $\mathcal{N} = 8$, $d = 4$ supergravities, reporting some recent advances on the moduli spaces associated to BPS and non-BPS attractor solutions supported by charge orbits with non-compact stabilizers.

The so-called *attractor mechanism* was first considered in the framework of $\mathcal{N}=2$, $d=4$ ungauged supergravity coupled to n_V vector multiplets [1]-[5]. It concerns the stabilization of the scalar fields ϕ^i ($i=1,...,n_V$) of the theory near the event horizon of an extremal, static, spherically symmetric and asymptotically flat black hole (BH) [6]. An extremal BH can be defined to have vanishing temperature ($T=0$), and thus it is thermodynamically stable. The asymptotical behavior of the scalars ϕ^i is defined by the limits

$$
\begin{aligned}
lim_{r\to\infty}\phi^i(r) &= \phi^i_\infty \in \mathcal{M}; && (1)\\
lim_{r\to r_H}\phi^i(r) &= \phi^i_H(q,p), && (2)
\end{aligned}
$$

where $\mathcal{M}$ is the scalar manifold, r_H is the radial coordinate of the event horizon, and (q,p) denotes the set $\left\{q_\Lambda, p^\Lambda\right\}$ of the electric and magnetic charges of the BH ($\Lambda=0,1,...,n_V$), which are conserved due to the overall $(U(1))^{n_V+1}$ gauge-invariance of the considered theory. The dynamical flow determining the radial evolution of the scalars $\phi^i(r)$ between the above two asymptotical limits is non-singular near the horizon, provided that

$$
\left.\frac{\partial V_{BH}(\phi,q,p)}{\partial\phi^i}\right|_{\phi^j=\phi^j_H}=0, \tag{3}
$$

where V_{BH} is a certain positive definite, charge-dependent function in $\mathcal{M}$, named *BH effective potential* [5]. The condition (3) determines the so-called *attractor equations*, whose solutions are the purely charge-dependent, stabilized horizon configurations $\phi^i_H(q,p)$ in the r.h.s. of Eq. (2). By using the Bekenstein-Hawking entropy-area formula [7, 5], the classical BH entropy reads

$$
\mathcal{S}(q,p)=\frac{A_H}{4}=\pi V_{BH}\left(\phi_H(q,p),q,p\right), \tag{4}
$$

where A_H is the area of the BH event horizon.

The horizon geometry of extremal, asymptotically flat BHs in $\mathcal{N}=2$, $d=4$ supergravity is a maximally supersymmetric $\mathcal{N}=2$ background, namely the Bertotti-Robinson (BR) $AdS_2\times S^2$ BH metric [9, 10], which in turn is a particular case of the extremal p-brane horizon geometry $AdS_{p+2}\times S^{d-p-2}$ [11].

The first class of attractors to be studied was the $\frac{1}{2}$-BPS one, which preserves 4 supersymmetries out of the 8 pertaining to the asymptotical $\mathcal{N}=2$, $d=4$ Poincaré superalgebra. Examples of such attractors are given by Figures 1 and 2. Recently, many important advances have been performed in the study of extremal BH attractors, mainly concerning new classes of attractor configurations, corresponding to non-BPS, non-supersymmetric horizon geometries [12]–[46].

For asymptotically flat extremal BHs V_{BH} is given in terms of the scalar-dependent, complex symmetric matrix $\mathcal{N}_{\Lambda\Sigma}(\phi)$ (with $Im\mathcal{N}_{\Lambda\Sigma}$ negative definite), determining the couplings of the Maxwell field strength terms $\mathcal{F}^2$ and $\mathcal{F}\widetilde{\mathcal{F}}$ in the Lagrangian density, and of the electric and magnetic BH charges [5]:

$$
V_{BH}(\phi,q,p)=-\frac{1}{2}\left(q_\Lambda-\mathcal{N}_{\Lambda\Sigma}p^\Sigma\right)\left(Im\mathcal{N}\right)^{-1|\Lambda\Delta}\left(q_\Delta-\overline{\mathcal{N}}_{\Delta\Gamma}p^\Gamma\right). \tag{5}
$$

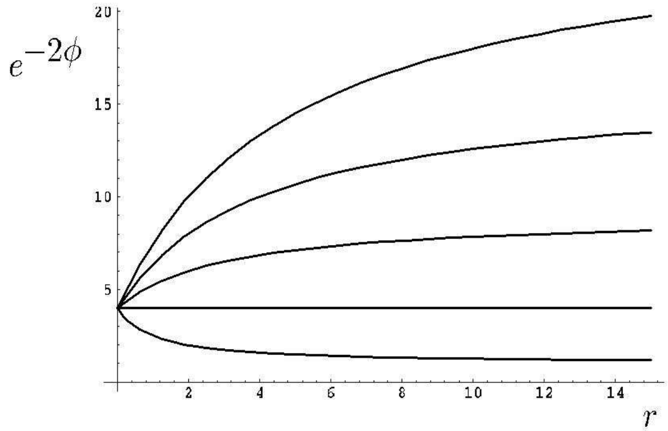

Figure 1: **Realization of the attractor mechanism in the $\frac{1}{2}$-BPS dilatonic BH [3, 4, 6]**. Independently on the set of asymptotical ($r \to \infty$) scalar configurations, the near-horizon evolution of the dilatonic function $e^{-2\phi}$ converges towards a fixed *attractor* value, which is purely dependent on the (ratio of the) quantized conserved charges of the BH.

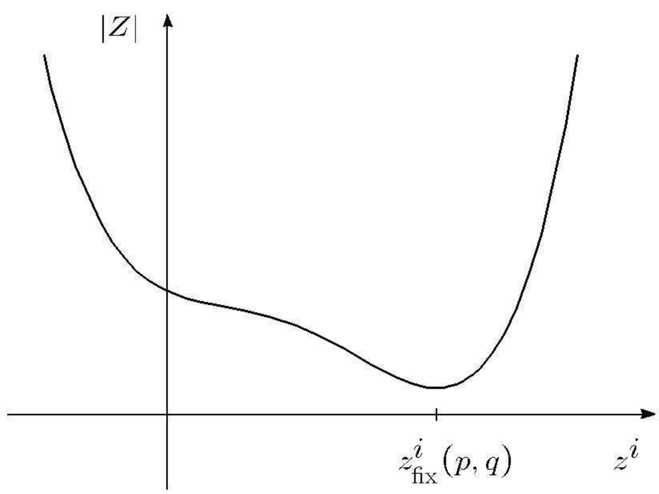

Figure 2: **Minimization of the absolute value of the "central charge" function Z in $\mathcal{M}$.** In the picture $z^i_{fix}(p,q)$ stands for the *attractor*, purely charge-dependent value of the scalars at the event horizon of the considered $\frac{1}{2}$-BPS extremal BH. The attractor mechanism fixes the extrema of the central charge to correspond to the discrete *fixed* points of the *attractor variety* [8] $\mathcal{M}$. Of course, the dependence of the central charge on scalars is shown for a given supporting BH charge configuration.

Such a formula is valid for any (not necessarily supersymmetric) theory coupling Einstein gravity to scalars and Maxwell vector fields, whose Lagrangian density in general has the form

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g}} &= -\frac{1}{2}R - g_{ab}\left(\partial_\mu \phi^a\right)\left(\partial_\nu \phi^b\right)G^{\mu\nu} + \\ &+ \left(Im\mathcal{N}_{\Lambda\Sigma}\right)\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma|\mu\nu} + \frac{1}{2\sqrt{-g}}\left(Re\mathcal{N}_{\Lambda\Sigma}\right)\epsilon^{\mu\nu\rho\lambda}\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma}_{\rho\lambda} + ..., \end{aligned} \tag{6}$$

where g_{ab} is the metric of the scalar manifold and $G_{\mu\nu}$ is the space-time metric.

An equivalent (but manifestly duality-covariant) expression reads [5]

$$V_{BH}\left(\phi, q, p\right) = -\frac{1}{2}Q^T M\left(\mathcal{N}\right)Q, \tag{7}$$

where Q^T is the $1 \times (2n_V + 2)$ vector $\left(p^\Lambda, q_\Lambda\right)$ of the BH charges, and $M\left(\mathcal{N}\right)$ is the symplectic $(2n_V + 2) \times (2n_V + 2)$ real, negative definite symmetric matrix

$$\begin{aligned} &M = R^T M_D R, \quad R \equiv \begin{pmatrix} I & 0 \\ -Re\mathcal{N} & I \end{pmatrix}, \quad M_D \equiv \begin{pmatrix} Im\mathcal{N} & 0 \\ 0 & \left(Im\mathcal{N}\right)^{-1} \end{pmatrix}, \\ &M\Omega M = \Omega, \ \Omega \equiv \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}. \end{aligned} \tag{8}$$

In $\mathcal{N} = 2$, $d = 4$ supergravity the scalar manifold is endowed with the so-called special Kähler geometry (see *e.g.* [47]) ; its metric $g_{i\bar{j}} = \partial_i \overline{\partial}_{\bar{j}} K$ (K being the Kähler potential) and $\mathcal{N}_{\Lambda\Sigma}$ are respectively given by the formulæ:

$$\begin{aligned} &g_{i\bar{j}} = -ie^K\left[\left(\overline{D}_{\bar{j}}\overline{X}^\Lambda\right)D_i F_\Lambda - \left(D_i X^\Lambda\right)\overline{D}_{\bar{j}}\overline{F}_\Lambda\right]; \\ &\mathcal{N}_{\Lambda\Sigma} = h_{I\Lambda}\left(f^{-1}\right)^I_\Sigma, \quad f^\Lambda_I \equiv e^{K/2}\left(X^\Lambda, \overline{D}_{\bar{i}}\overline{X}^\Lambda\right), \quad h_{I\Lambda} \equiv e^{K/2}\left(F_\Lambda, \overline{D}_{\bar{i}}\overline{F}_\Lambda\right), \end{aligned} \tag{9}$$

where D_i denotes the Kähler-covariant derivative, and $\left(X^\Lambda, F_\Lambda\right)$ are the holomorphic sections of the Hodge bundle over $\mathcal{M}$ ($F_\Lambda = \frac{\partial F(X)}{\partial X^\Lambda}$, whenever the holomorphic prepotential function $F\left(X\right)$ exists) (see *e.g.* [47] and Refs. therein).

The symplectic-covariant formulation of the $\mathcal{N} = 2$ special Kähler geometry can be actually generalized to all extended ($\mathcal{N} = 3, ..., 8$) $d = 4$ supergravities [48, 25, 49]. In such theories, V_{BH} can be expressed as

$$V_{BH} = \frac{1}{2}\left|Z_{AB}\right|^2 + \left|Z^I\right|^2, \tag{10}$$

where Z_{AB} $(A, B = 1, ..., \mathcal{N})$ is the antisymmetric *central charge matrix* and Z^I are the so-called *dressed* (or *matter*) *charges*, respectively appearing in the supersymmetry transformations of the gravitinos $\psi_{\mu A}$ and of the other fermions λ^I_A of the theory in the considered BH background:

$$\begin{aligned} &\delta_\varepsilon \psi_{\mu A}\big|_{BH} \sim Z_{AB}\gamma_\mu \varepsilon^B; \\ &\delta_\varepsilon \lambda^I_A\big|_{BH} \sim Z^I \varepsilon_A, \end{aligned} \tag{11}$$

where γ_μ are the γ-matrices and ε is the parameter of the supersymmetric transformation.

Let us consider the maximal $d = 4$ supergravity, *i.e.* $\mathcal{N} = 8$ supergravity, based on the real 70-dim. symmetric manifold $\frac{E_{7(7)}}{SU(8)}$ [50]. In this case no matter multiplets are coupled to the gravity one, thus Eq. (10) simplifies to $(A, B = 1, ..., 8)$

$$V_{BH} = \frac{1}{2} \left| Z_{AB} \right|^2 , \tag{12}$$

with $Z_{AB} = L^{\Lambda}_{AB} (\phi) Q_{\Lambda}$, where $L(\phi) \in E_{7(7)}$ and Q is the charge vector. Under a transformation h of the stabilizer $SU(8)$, the matrix Z transforms as [42]

$$Z(\phi, Q) \longmapsto Z(\phi_g, Q) = hZ\left(\phi_g, g^{-1}Q\right) \Longrightarrow V_{BH}(\phi, Q) = V_{BH}\left(\phi_g, g^{-1}Q\right). \tag{13}$$

By computing V_{BH} at one of its critical points, one obtains a completely charge-dependent expression:

$$V_{BH}|_{\frac{\partial V_{BH}}{\partial \phi}=0} \equiv V_{BH,cr.}(Q) = V_{BH,cr.}\left(g^{-1}Q\right) \sim \sqrt{|\mathcal{J}_4|}, \tag{14}$$

$\mathcal{J}_4$ being the quartic Cartan-Cremmer-Julia invariant of the fundamental representation **56** of $E_{7(7)}$ [50, 51].

The local $SU(8)$ symmetry allows one to go to the so-called "normal frame" [52]. In such a frame, Z_{AB} and $\mathcal{J}_4$ respectively read as follows:

$$Z_{AB,normal} = skew - diag\,(\rho_1, \rho_2, \rho_3, \rho_4)\, e^{i\varphi/4};$$

$$\mathcal{J}_{4,normal} = \left[(\rho_1 + \rho_2)^2 - (\rho_3 + \rho_4)^2\right]\left[(\rho_1 - \rho_2)^2 - (\rho_3 - \rho_4)^2\right] + 8\rho_1\rho_2\rho_3\rho_4(cos\varphi - 1), \tag{15}$$

with $\rho_i \in R^+ \; \forall i = 1, ..., 4$. Note that $Z_{AB,normal}$ has an $(SU(2))^4$ symmetry.

From the analysis performed in [53, 54, 25], the $\mathcal{N} = 8$ attractor equations yield only 2 distinct classes of solutions with non-vanishing entropy ($\frac{1}{8}$-BPS for $\mathcal{J}_4 > 0$, non-BPS for $\mathcal{J}_4 < 0$):

1. $\frac{1}{8}$-BPS: $\rho_1 = \rho_{\frac{1}{8}-BPS} \in R_0^+$ and all the others vanish, $\mathcal{J}_{4,normal,\frac{1}{8}-BPS} > 0$, and

$$\mathcal{S}_{\frac{1}{8}-BPS} = \pi\sqrt{\mathcal{J}_{4,normal,\frac{1}{8}-BPS}} = \pi\rho_1^2. \tag{16}$$

The corresponding orbit of supporting BH charges in the **56** of $E_{7(7)}$ is $\mathcal{O}_{\frac{1}{8}-BPS} = \frac{E_{7(7)}}{E_{6(2)}}$. Moreover, $Z_{AB,normal,\frac{1}{8}-BPS}$ has symmetry enhancement (*m.c.s.* stands for *maximal compact subgroup*)

$$(SU(2))^4 \longrightarrow SU(6) \otimes SU(2) = m.c.s.\left(E_{6(2)}\right). \tag{17}$$

Notice that $\varphi_{\frac{1}{8}-BPS}$ is actually undetermined.

2. non-BPS: all ρs are equal to $\rho_{non-BPS} \in R_0^+$, $\varphi_{non-BPS} = \pi$, $\mathcal{J}_{4,normal,non-BPS} < 0$, and

$$\mathcal{S}_{non-BPS} = \pi\sqrt{-\mathcal{J}_{4,normal,non-BPS}} = 4\pi\rho^2. \tag{18}$$

The corresponding orbit of supporting BH charges in the **56** of $E_{7(7)}$ is $\mathcal{O}_{non-BPS} = \frac{E_{7(7)}}{E_{6(6)}}$. Furthermore, $Z_{AB,normal,non-BPS}$ has symmetry enhancement

$$(SU(2))^4 \longrightarrow USp(8) = m.c.s.\left(E_{6(6)}\right). \tag{19}$$

Thus, the symmetry of $Z_{AB,normal}$ gets enhanced at the particular points of $\frac{E_{7(7)}}{SU(8)}$ given by the solutions of $\mathcal{N}=8$, $d=4$ attractor equations with non-vanishing $\mathcal{J}_4$. In general, *the invariance properties of the solutions to attractor Eqs. with $\mathcal{J}_4 \neq 0$ are given by the m.c.s. of the stabilizer of the corresponding supporting BH charge orbit.*

The 70×70 Hessian matrix of V_{BH} at the $\frac{1}{8}$-BPS critical points has rank 30; its 30 strictly positive and 40 vanishing eigenvalues respectively correspond to the 15 vector multiplets and to the 10 hypermultiplets of the $\mathcal{N}=2$, $d=4$ spectrum obtained by reducing $\mathcal{N}=8$ supergravity according to the following branching of the **70** (four-fold antisymmetric) of $SU(8)$ [55]:

$$\begin{aligned} & SU(8) \longrightarrow SU(6) \otimes SU(2); \\ & \mathbf{70} \longrightarrow \left[(\mathbf{15},\mathbf{1}) \oplus \left(\overline{\mathbf{15}},\mathbf{1}\right) \right]_{m\neq 0} \oplus (\mathbf{20},\mathbf{2})_{m=0}. \end{aligned} \tag{20}$$

On the other hand, at the non-BPS critical points the Hessian matrix has rank 28; such a splitting of the mass spectrum can be interpreted according to the following branching of the **70** of $SU(8)$ [39]:

$$\begin{aligned} & SU(8) \longrightarrow USp(8); \\ & \mathbf{70} \longrightarrow (\mathbf{1} \oplus \mathbf{27})_{m\neq 0} \oplus (\mathbf{42})_{m=0}. \end{aligned} \tag{21}$$

As shown in [42], the massless modes of the critical Hessian matrix actually correspond to flat directions of V_{BH} itself. This can be easily realized by noticing that the stabilizers of the charge orbits are non-compact, so that

$$\begin{aligned} & g_Q Q^{BPS} = Q^{BPS}, \ \forall g_Q \in E_{6(2)}; \\ & g_Q Q^{non-BPS} = Q^{non-BPS}, \ \forall g_Q \in E_{6(6)}, \end{aligned} \tag{22}$$

and thus at the critical points (recall Eq. (13))

$$V_{BH}\left(\phi_{g_Q}, g_Q^{-1} Q\right) = V_{BH}\left(\phi_{g_Q}, Q\right) = V_{BH}\left(\phi, Q\right). \tag{23}$$

This implies that each of the two classes of $\mathcal{N}=8$, $d=4$ extremal BH attractors with non-vanishing entropy has an associated moduli space:

$$\begin{aligned} & BPS: \frac{E_{6(2)}}{SU(6)\otimes SU(2)}, \ quaternionic \ manifold \ with \ dim_R = 40; \\ & non-BPS: \frac{E_{6(6)}}{USp(8)}, \ \mathcal{N}=8, \ d=5 \ scalar \ manifold \ with \ dim_R = 42. \end{aligned} \tag{24}$$

The same reasoning, which is actually independent on the number d of space-time dimensions and on $\mathcal{N}$, will apply to *all* theories of the kind considered above, whose scalar manifold is an homogeneous (not necessarily symmetric) space, when the stabilizer of the orbit of the attractor-supporting charge vector Q is non-compact [42]. For $\mathcal{N}>2$ this will apply to both BPS and non-BPS critical points (as shown above for $\mathcal{N}=8$, $d=4$). However, for $\mathcal{N}=2$, $d=4$ the stabilizer of the ($\frac{1}{2}$-)BPS orbit is compact, and no flat directions will occur (apart from hypermultiplets). This is strictly true as far as the

metric of the scalar manifold is strictly positive definite at the considered BPS critical points. Indeed, by using special Kähler geometry one can prove the following result, holding for any $\mathcal{N}=2$, $d=4$ supergravity [5] (such a result, *mutatis mutandis*, holds also for $d=5$ [56]):

$$\left(D_i\overline{D}_{\overline{j}}V_{BH}\right)_{BPS}=2\left(g_{i\overline{j}}V_{BH}\right)_{BPS}. \tag{25}$$

Reconsidering $\mathcal{N}=2$, $d=4$ supergravity, the Riemann tensor of the special Kähler scalar manifold satisfies the following relation (see *e.g.* [47] and Refs. therein)

$$R_{i\overline{j}l\overline{k}}=-g_{i\overline{j}}g_{l\overline{k}}-g_{i\overline{k}}g_{l\overline{j}}+C_{ilp}\overline{C}_{\overline{j}\overline{k}\overline{p}}g^{p\overline{p}}, \tag{26}$$

where the rank-3 completely symmetric tensor C_{ijk} has the properties

$$\overline{D}_{\overline{l}}C_{ijk}=0,\ \ D_{[l}C_{i]jk}=0. \tag{27}$$

In particular, for homogeneous symmetric cubic special Kähler geometries another set of relations holds [57, 58] (see also [49], [34] and Refs. therein; here and below z^i denote the complex scalars):

$$\begin{aligned}
&D_lC_{ijk}=0;\\
&C_{ijk}=e^K\partial_i\partial_j\partial_k f\left(z\right),\quad f\left(z\right)\equiv\tfrac{1}{3!}d_{ijk}z^iz^jz^k;\\
&\overline{E}_{\overline{i}ijpq}\equiv g^{k\overline{k}}g^{r\overline{j}}C_{r(pq}C_{ij)k}\overline{C}_{\overline{k}\overline{i}\overline{j}}-\tfrac{4}{3}g_{(q|\overline{i}}C_{|ijp)}=0;\\
&d_{ABC}d^{B(PQ}d^{LM)C}=\tfrac{4}{3}\delta_A^{(P}d^{QLM)}.
\end{aligned} \tag{28}$$

The $\mathcal{N}=2$, $d=4$ attractor equations read [5]

$$2\overline{Z}D_iZ+iC_{ijk}g^{j\overline{j}}g^{k\overline{k}}\left(\overline{D}_{\overline{j}}\overline{Z}\right)\overline{D}_{\overline{k}}\overline{Z}=0, \tag{29}$$

Z denoting the $\mathcal{N}=2$ covariantly holomorphic *central charge function* [47]

$$Z\equiv e^{K/2}\left(X^\Lambda q_\Lambda-F_\Lambda p^\Lambda\right). \tag{30}$$

Eqs. (29) yield three classes of solutions [23, 27]:

i) $\frac{1}{2}$-BPS solutions [5], with $Z\neq 0$ and $D_iZ=0\ \forall i$. They saturate the BPS bound [59]:

$$M^2_{ADM,BPS}=\left|Z\right|^2_{BPS}, \tag{31}$$

M_{ADM} being the Arnowitt-Deser-Misner BH mass [60].

ii) non-BPS solutions with $Z\neq 0$ and $D_iZ\neq 0$ for at least some i [5, 13, 15, 16, 23]. They do not preserve any supersymmetry and do not saturate the BPS bound; indeed, for symmetric spaces it holds that [27]:

$$M^2_{ADM,non-BPS,Z\neq 0}=4\left|Z\right|^2_{non-BPS,Z\neq 0}>\left|Z\right|^2_{non-BPS,Z\neq 0}. \tag{32}$$

Such a result actually holds for homogeneous non-symmetric [34] and also for generic cubic (at least within some particular assumptions [16]) special Kähler geometries.

iii) non-BPS solutions with $Z=0$ and $D_iZ \neq 0$ for at least some i [24, 27, 34, 46]. They do not preserve any supersymmetry and do not saturate the BPS bound:

$$M^2_{ADM,non-BPS,Z=0} = \left[g^{i\overline{j}} \left(D_i Z \right) \overline{D}_{\overline{j}} \overline{Z} \right]_{non-BPS,Z=0} > 0. \tag{33}$$

As mentioned above, $\frac{1}{2}$-BPS critical points are stable; they have no massless Hessian modes at all, and thus they do not have any associated moduli space. The moduli spaces associated to the $\mathcal{N}=2$, $d=4$ non-BPS solutions with $Z \neq 0$ and $Z=0$ and to the $\mathcal{N}=2$, $d=5$ non-BPS solutions have been recently determined in [42] (see also [45]); they are respectively given by Tables 2, 3 and 4 of [42].

As obtained in [16], the $2n_V \times 2n_V$ (real form of the) Hessian matrix of V_{BH} at its non-BPS $Z \neq 0$ critical points in a generic cubic special Kähler geometry of complex dimension $dim_C = n_V$ has $n_V + 1$ strictly positive and $n_V - 1$ vanishing eigenvalues. As pointed out above, in the homogeneous (not necessarily symmetric) case, these latter $n_V - 1$ massless Hessian modes actually correspond to $n_V - 1$ flat directions of $V_{BH,non-BPS,Z\neq 0}$ [42].

The same result holds also for generic cubic special Kähler geometries, at least for some particular BH charge configurations [45]. This is simply seen *e.g.* by splitting the complex scalars as $z^i = x^i - i\lambda^i$, and considering the peculiar non-BPS $Z \neq 0$-supporting BH charge configuration $Q_0^T = (p^0, 0, q_0, 0)$, for which the criticality conditions $\frac{\partial V_{BH}}{\partial x^i} = 0$ can be solved by putting $x^i = 0 \ \forall i$. For such a case, in [45] V_{BH} was shown to acquire the following simple form:

$$\left. V_{BH} \right|_{x^i=0 \ \forall i, \ Q=Q_0} = \frac{1}{2} \left[\left(p^0 \right)^2 \mathcal{V} + \left(q_0 \right)^2 \mathcal{V}^{-1} \right] \equiv V^*_{BH} \left(\mathcal{V}, p^0, q_0 \right), \tag{34}$$

where $\mathcal{V} \equiv \frac{1}{3!} d_{ijk} \lambda^i \lambda^j \lambda^k$. By rescaling $\lambda^i \equiv \mathcal{V}^{1/3} \widehat{\lambda}^i$, it is immediate to realize that $V^*_{BH} \left(\mathcal{V}, p^0, q_0 \right)$ *does not depend on any of the* $\widehat{\lambda}^i$. By definition, the $\widehat{\lambda}^i$s belong to the geometrical *locus* $\frac{1}{3!} d_{ijk} \widehat{\lambda}^i \widehat{\lambda}^j \widehat{\lambda}^k = 1$; thus, they parameterize $n_V - 1$ "flat" directions of $\left. V_{BH} \right|_{x^i=0 \ \forall i}$ at its non-BPS $Z \neq 0$ critical points supported by the charge configuration Q_0. Such $n_V - 1$ "flat" directions turn out to span nothing but the $(n_V - 1)$-dim. real special scalar manifold of the corresponding $\mathcal{N}=2$, $d=5$ parent supergravity theory [45].

Let us now consider an explicit example, namely the *magic* $\mathcal{N}=2$, $d=4$ supergravity theory based on the exceptional Jordan algebra J_3^{O} over the octonions (see *e.g.* [27, 61, 39, 42] and Refs. therein). It is based on the rank-3 homogeneous symmetric special Kähler manifold $\frac{E_{7(-25)}}{E_{6(-78)} \otimes U(1)}$ with $dim_C = n_V = 27$; the charge vector Q sits in the fundamental representation $\mathbf{56}$ of $E_{7(-25)}$.

The $\frac{1}{2}$-BPS attractors are supported by a Q belonging to the BPS orbit $\frac{E_{7(-25)}}{E_{6(-78)}}$ ($dim_R = 55$); due to the compactness of $E_{6(-78)}$, there is no BPS moduli space at all.

On the other hand, the non-BPS $Z \neq 0$ attractors are supported by a Q belonging to the 55-dim. non-BPS orbit $\frac{E_{7(-25)}}{E_{6(-26)}}$, $E_{6(-26)}$ being a non-compact real form of the exceptional group E_6; the corresponding non-BPS $Z \neq 0$ moduli space reads $\frac{E_{6(-26)}}{F_{4(-52)}}$ ($dim_R = 26$), where $F_{4(-52)} = m.c.s. \left(E_{6(-26)} \right)$ [62]. It is nothing but the rank-2, real special scalar manifold of the corresponding $\mathcal{N}=2$, $d=5$ parent supergravity theory [45].

The non-BPS $Z=0$ attractors are supported by a Q belonging to the 55-dim. non-BPS orbit $\frac{E_{7(-25)}}{E_{6(-14)}}$, $E_{6(-14)}$ being the only other non-compact real form of E_6 contained in $E_{7(-25)}$ [62]; the corresponding non-BPS $Z=0$ moduli space is the rank-2, homogeneous symmetric (not special) Kähler manifold $\frac{E_{6(-14)}}{SO(10)\otimes U(1)}$ ($dim_C=16$) [45], where $SO(10)\otimes U(1)=m.c.s.\left(E_{6(-14)}\right)$ [62].

The corresponding parent theory in $d=5$ is the *magic* $\mathcal{N}=2$, $d=5$ supergravity over J_3^O (see *e.g.* [57, 54, 42] and Refs. therein). For such a theory, the BPS charge orbit coincides with $\frac{E_{6(-26)}}{F_{4(-52)}}$ itself [54], and there are no BPS massless Hessian modes [56]. The unique class of non-BPS attractors with non-vanishing cubic invariant I_3 (see *e.g.* [56] and Refs. therein) is supported by the 26-dim. BH charge orbit $\frac{E_{6(-26)}}{F_{4(-20)}}$, $F_{4(-20)}$ being the only non-compact real form of the exceptional group F_4 contained in $E_{6(-26)}$ [62]. The corresponding non-BPS moduli space is the rank-1, homogeneous symmetric manifold $\frac{F_{4(-20)}}{SO(9)}$ ($dim_R=16$) [42], where $SO(9)=m.c.s.\left(F_{4(-20)}\right)$ [62].

Finally, it is worth remarking that the non-BPS $d=5$ attractors can give rise to both $Z\neq 0$ and $Z=0$ non-BPS $d=4$ critical points, depending on the sign of an extra Kaluza-Klein charge [45]. This implies that the moduli space of non-BPS $d=5$ attractors is contained in the moduli spaces of both species ($Z\neq 0$ and $Z=0$) of non-BPS $d=4$ attractors, as pointed out in [45] (and as given by the Tables 2, 3 and 4 of [42]).

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